Pole Locations

Pole locations

Pole locations of $F(s)$ affect qualitative behavior of $f(t)$

For this discussion, consider *rational functions* $F(s)$

i.e. $F(s) = \frac{N(s)}{D(s)}$, where $N(s), D(s)$ are polynomials in s

Ex. $\frac{1}{s}$, $\frac{1}{s^2}$, $\frac{s}{s^2+1}$

These are the Laplace transform of 1, t, $\cos t$

An example of a non-rational Laplace transform is e^{-s}

Pole locations

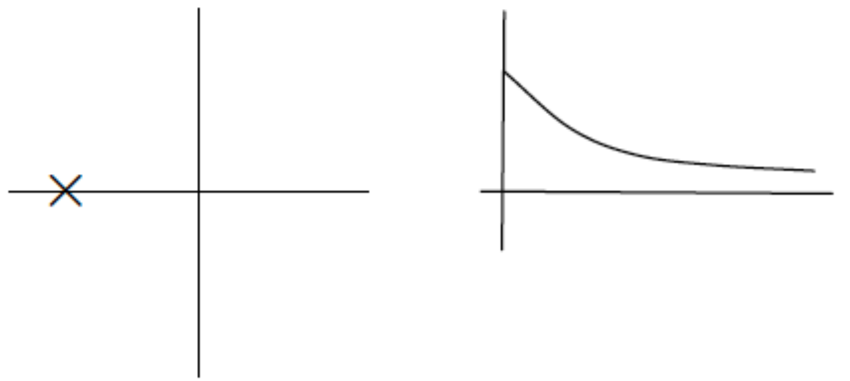
Consider a rational functions $F(s) = \frac{N(s)}{D(s)}$.

The *poles* of $F(s)$ are the values of s such that $D(s) = 0$

The locations of the poles on the complex plane provides information about the behavior of $f(t)$

Example

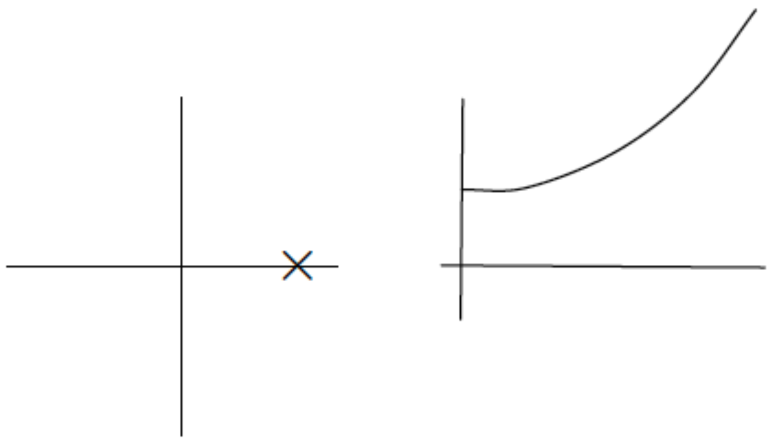
$f(t) = \mathrm{e}^{-t}$, $F(s) = \frac{1}{s+1}$, pole $= -1$



Observation: a single negative pole corresponds to a decaying exponential in the time domain

Example

$f(t) = e^t$, $F(s) = \frac{1}{s-1}$, pole $= 1$



Observation: a single positive pole corresponds to a blowing-up exponential in the time domain

Example

$f(t) = 1,\ F(s) = \frac{1}{s}$, pole $= 0$

Observation: a single pole $s = 0$ corresponds to a constant in the time domain

Pole locations

1. A single real negative pole corresponds to a decaying exponential. The farther left the pole is, the faster $f(t)$ decays.

2. A single real positive pole corresponds to a blowing-up exponential. The farther right the pole is, the faster $f(t)$ blows up.

3. A single pole $s = 0$ corresponds to a constant in the time domain.

Example

$f(t) = \sin t, \ F(s) = \frac{1}{s^2+1}, \ \text{pole} = \pm j$

Observation: a complex conjugate pair of poles with $\text{Re}(s) = 0$ corresponds to a sinusoid with constant amplitude in the time domain

Example

$f(t) = \mathrm{e}^{-t} \sin t$, $F(s) = \frac{1}{(s+1)^2+1}$, pole $= -1 \pm j$

Observation: a complex conjugate pair of poles
with $\mathrm{Re}(s) < 0$ corresponds to a sinusoid
with exponentially decaying amplitude in the time domain

Example

$f(t) = \mathrm{e}^t \sin t,\ F(s) = \frac{1}{(s-1)^2+1},\ \text{pole} = 1 \pm j$

Observation: a complex conjugate pair of poles
with $\mathrm{Re}(s) > 0$ corresponds to a sinusoid
with exponentially blowing-up amplitude in the time domain

Pole locations

1. A complex conjugate pair of poles with $\mathrm{Re}(s) = 0$ corresponds to a sinusoid with constant amplitude.

2. A complex conjugate pair of poles with $\mathrm{Re}(s) < 0$ corresponds to a sinusoid with exponentially decaying amplitude.

3. A complex conjugate pair of poles with $\mathrm{Re}(s) > 0$ corresponds to a sinusoid with exponentially blowing-up amplitude.

Pole locations

1. A double real negative pole corresponds to an amplitude-modified decaying amplitude.

$$\text{Ex. } F(s) = \frac{1}{(s+1)^2},\ f(t) = t\mathrm{e}^{-t}$$

2. A double real positive pole corresponds to an amplitude-modified blowing-up amplitude.

$$\text{Ex. } F(s) = \frac{1}{(s-1)^2},\ f(t) = t\mathrm{e}^{t}$$

3. A double complex conjugate pair of poles with $\mathrm{Re}(s) = 0$ corresponds to a sinusoid with ramp-like amplitude.

$$\text{Ex. } F(s) = \frac{1}{(s^2+1)^2},\ f(t) = \frac{1}{2}(\sin t - t\cos t)$$

Pole locations

Note: for a control engineer, the left half-plane is "good", and the right half-plane is "bad"

Note: pole locations of $F(s)$ are a good indicator of only the *qualitative* behavior of $f(t)$, but not the *quantitative*

Ex. $F(s)$ has poles -10, $-2 \pm 10j$

$10j$ part may suggest severe oscillations in $f(t)$

but $f(t)$ can be $f(t) = 50\mathrm{e}^{-10t} + 0.001\mathrm{e}^{-2t}\cos(10t)$