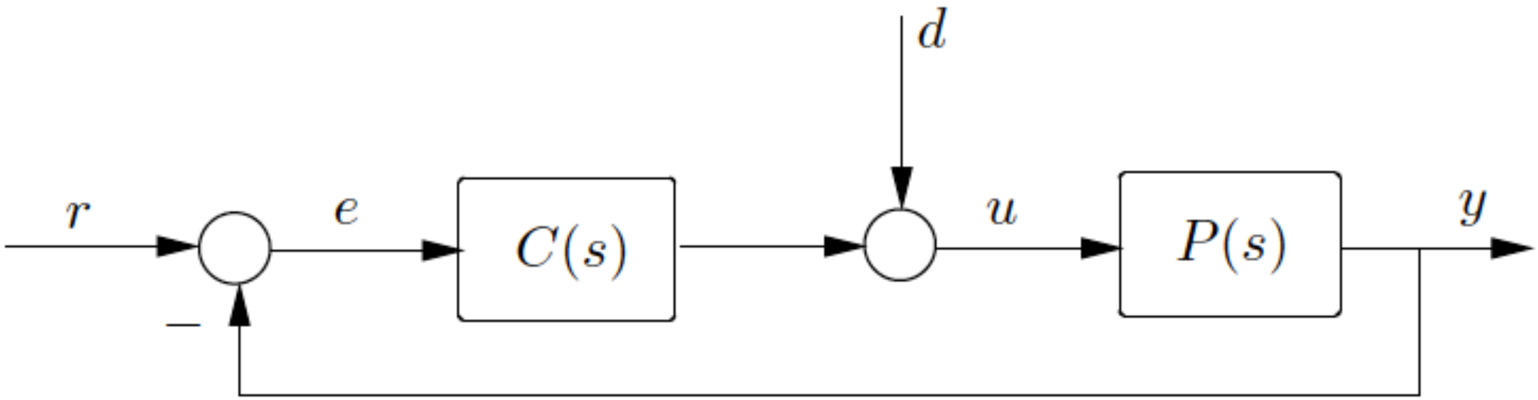
Stability Margins on Bode Plots

Phase/gain/stability margins



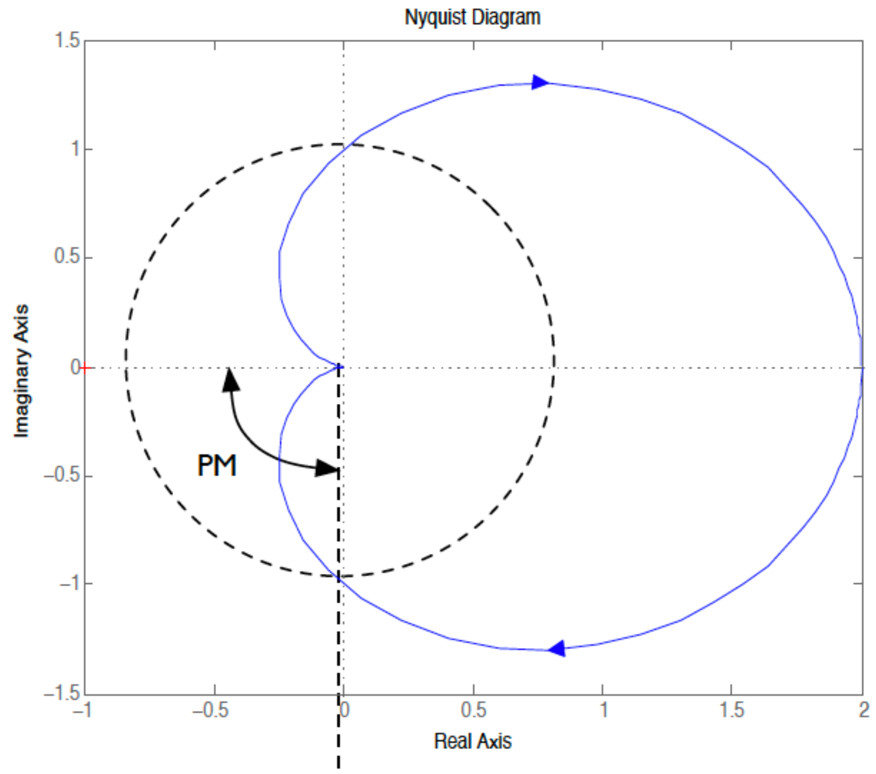
Suppose the feedback loop is stable; how stable is it?

This can be measured by magnititude and phase of $P(s)C(s)$

Three measures defined on Nyquist plot:
phase margin (PM), gain margin (GM), stability margin (SM)

Example

$P(s)C(s) = \frac{2}{(s+1)^2}$ $(K = 1)$
(assume no right half-plane pole-zero cancelations)



From Nyquist plot:

PM$= -180 - \angle P(j\omega_{gc})C(j\omega_{gc})$,
where ω_{gc} is s.t. $|P(j\omega_{gc})C(j\omega_{gc})| = 1$

GM$= -20\log_{10}|P(j\omega_{pc})C(j\omega_{pc})|$,
where ω_{pc} is s.t. $\angle P(j\omega_{pc})C(j\omega_{pc}) = -180$

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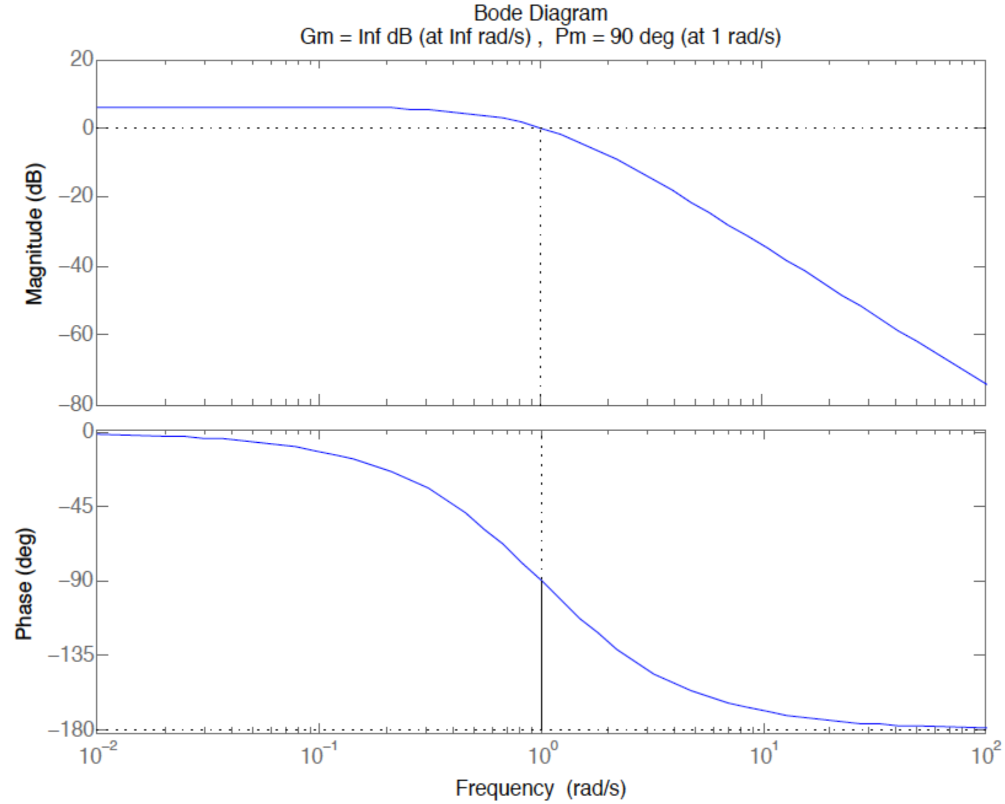

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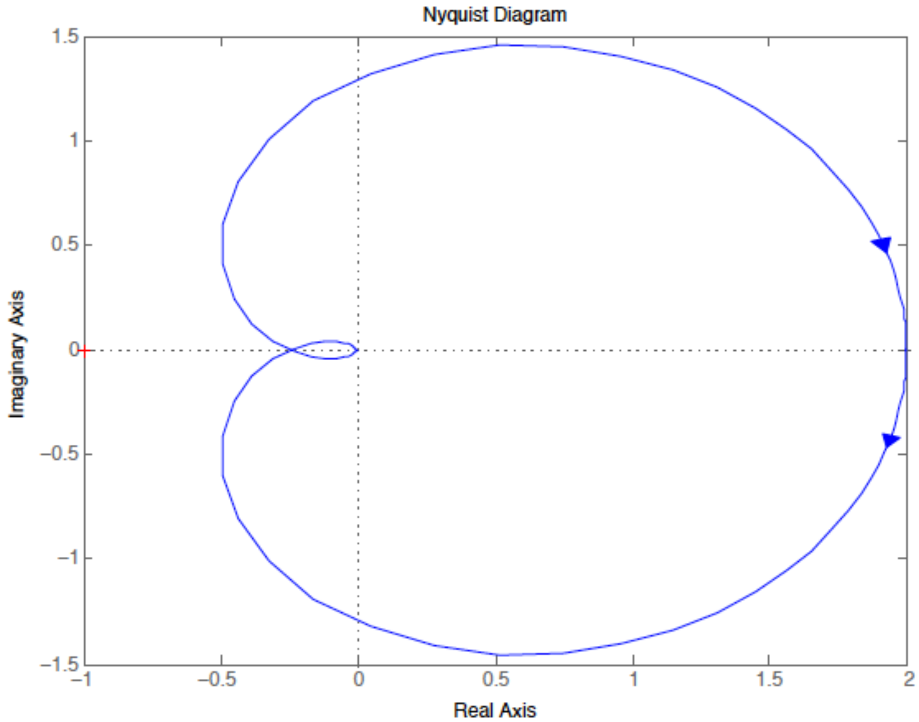

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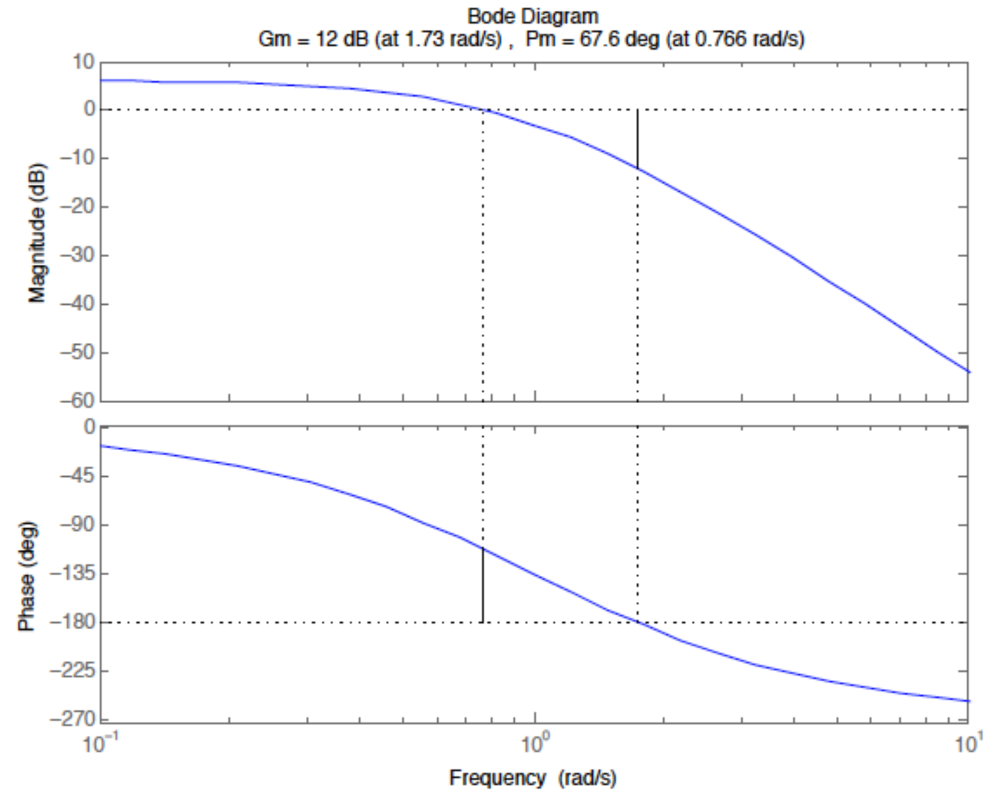

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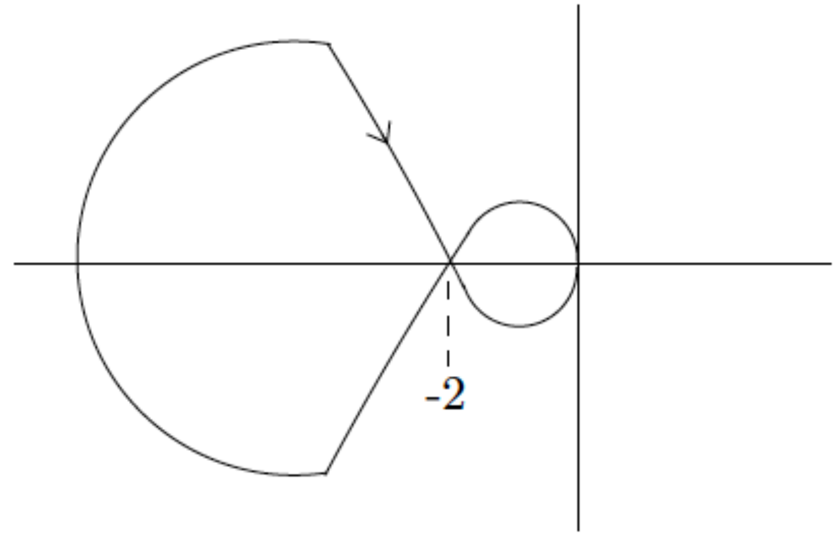

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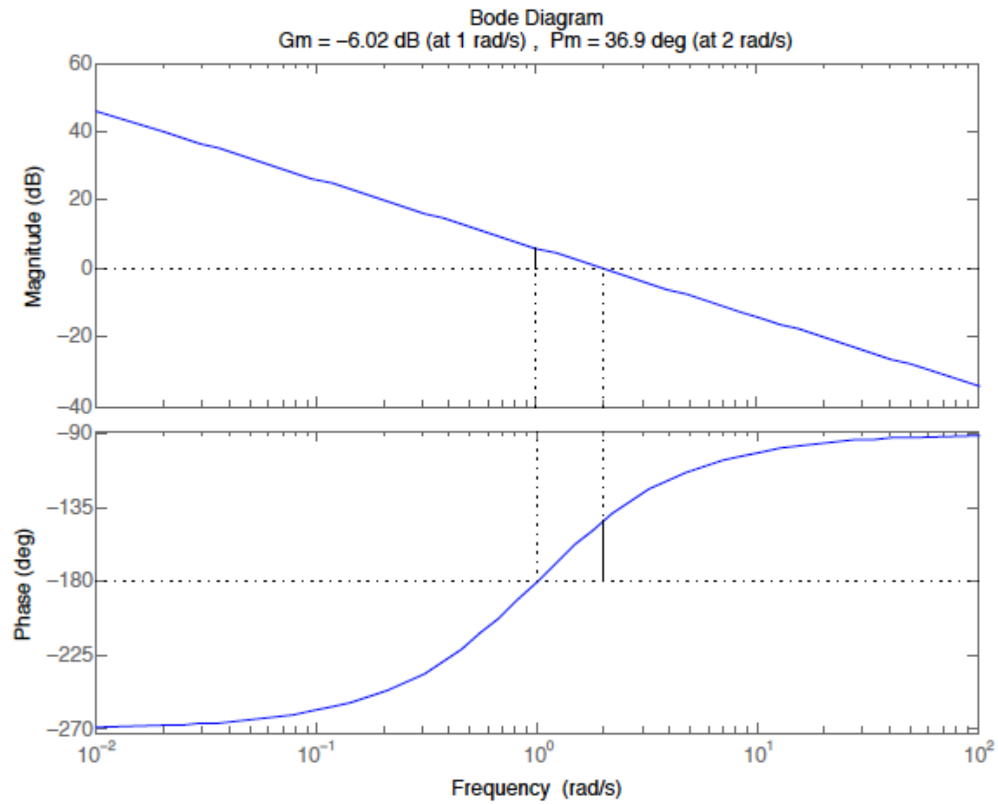

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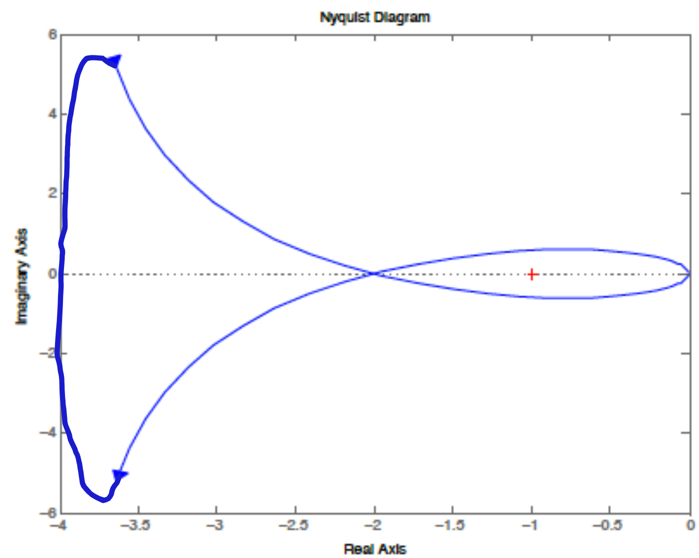
Stability margin

Stability margin is the distance from the critical point -1 to the closest point on the Nyquist plot



$$\text{SM} = \min_\omega |-\tfrac{1}{K} - P(j\omega)C(j\omega)|$$
$$= \min_\omega |\tfrac{1}{K} + P(j\omega)C(j\omega)|$$

Consider $K = 1$. Thus

$$\frac{1}{\text{SM}} = \max_\omega \frac{1}{|1+P(j\omega)C(j\omega)|}$$

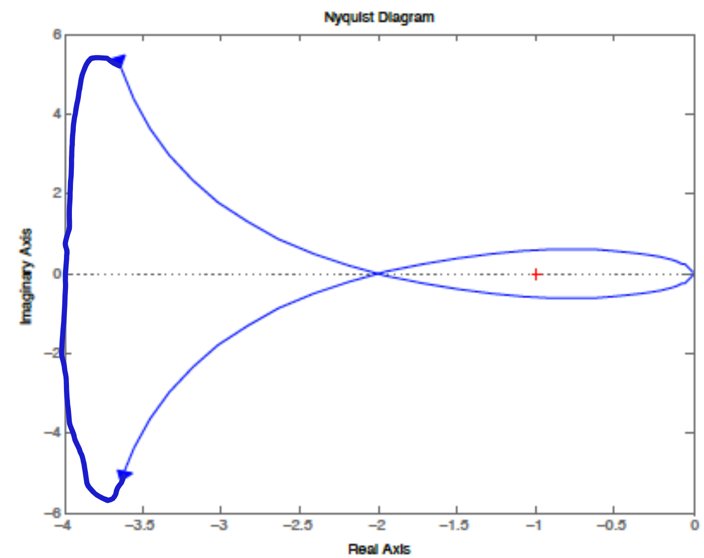
Draw Bode plots of transfer function $S(s) = \frac{1}{1+P(s)C(s)}$

Then the maximum height of the magnitude plot is $20\log_{10}\frac{1}{\text{SM}}$

Example

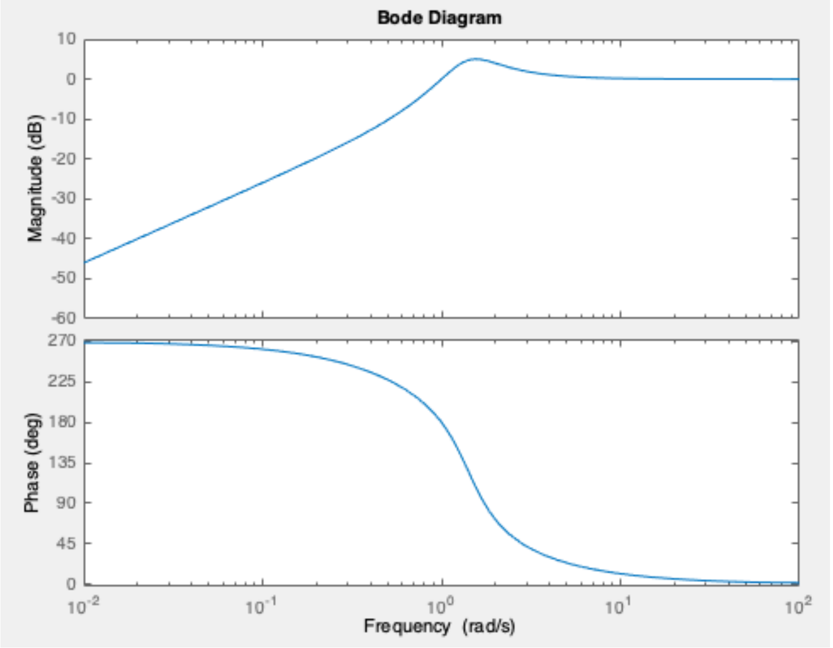
$P(s)C(s) = \frac{2(s+1)}{s(s-1)}$ $(K = 1)$
(assume no right half-plane pole-zero cancelations)

We've seen SM$= 0.57$



Bode plots of $\frac{1}{1+P(s)C(s)}$

Maximum magnitude is 5.02
(so $20\log_{10}\frac{1}{\text{SM}} = 5.02$)



Therefore SM $=$