Bode Plots

Intuition

Given $P(s)$ and $KC(s)$, Nyquist plot enables us to *analyze* the stability of the feedback loop

It is fine, if stability is confirmed

What if the feedback loop is determined unstable (no matter how we change the value of K)?

We need to somehow improve $C(s)$, this is *design*

Nyquist plot is convenient for analysis, but not for design

Bode plots

Consider a transfer function $G(s)$

Bode plots of $G(s)$ refer to two plots:

1) magnitude of $G(j\omega)$ versus ω
verticle: $20\log_{10}|G(j\omega)|$ (dB) on linear scale
horizontal: ω radians/second on logarithmic scale

2) angle of $G(j\omega)$ versus ω
verticle: $\angle G(j\omega)$ degrees on linear scale
horizontal: ω radians/second on logarithmic scale

Example

$$G(s) = Ks^n \ (K, n \text{ positive integer})$$

$$20\log_{10}|G(jw)| = 20\log_{10}|K(j\omega)^n| =$$

$$\angle G(jw) = \angle K(j\omega)^n =$$

Example

$$G(s) = Ks^n \ (K, n \text{ positive integer})$$

$$20\log_{10}|G(jw)| = 20\log_{10}|K(j\omega)^n| = 20\log_{10}K + 20n\log_{10}\omega$$

$$\angle G(jw) = \angle K(j\omega)^n = 90n$$

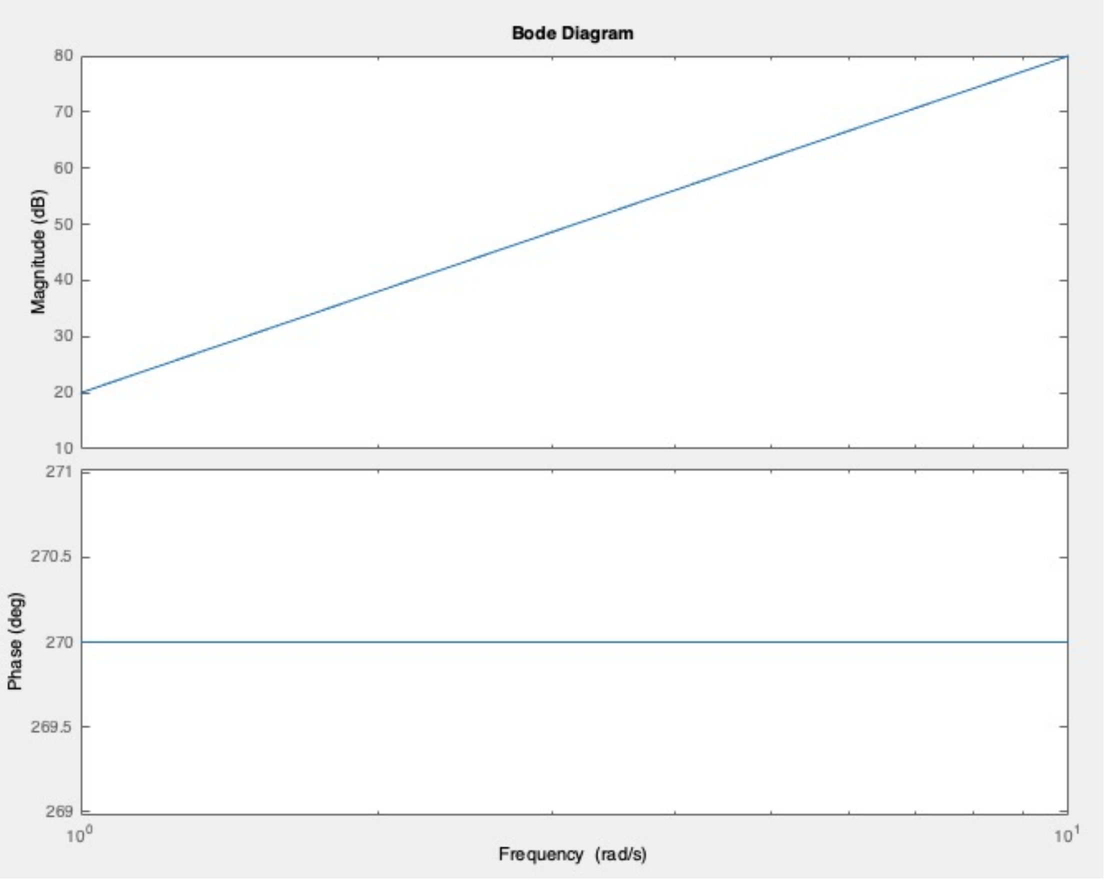
$n = 3$

$K = 10$

s=tf('s')

G=s^n

bode(G)



Example

$G(s) = \frac{K}{s^n}$ (K, n positive integer)

$20\log_{10}|G(jw)| = 20\log_{10}|\frac{K}{(j\omega)^n}| =$

$\angle G(jw) = \angle\frac{K}{(j\omega)^n} =$

Example

$G(s) = \frac{K}{s^n}$ (K, n positive integer)

$20\log_{10}|G(jw)| = 20\log_{10}|\frac{K}{(j\omega)^n}| = 20\log_{10}K - 20n\log_{10}\omega$

$\angle G(jw) = \angle\frac{K}{(j\omega)^n} = -90n$

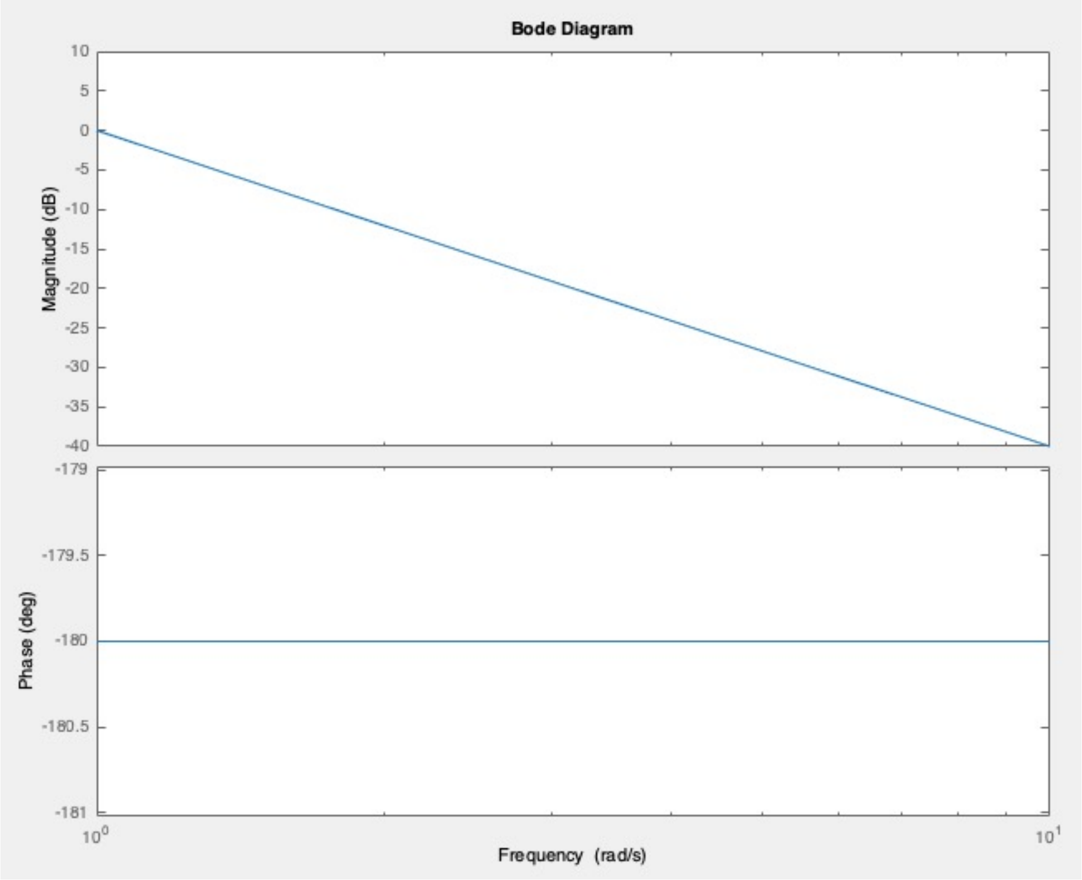
$n = 2$

$K = 1$

s=tf('s')

G=s^n

bode(G)



Example

$G(s) = Ts \pm 1$ (T positive)

$20\log_{10}|G(jw)| = 20\log_{10}|Tj\omega \pm 1|$

At low frequency $= 20\log_{10}|\pm 1| =$

At high frequency $= 20\log_{10}|Tj\omega| =$

Two lines meet at $\omega = \frac{1}{T}$ (break point)

Example

$G(s) = Ts \pm 1$ (T positive)

$\angle G(jw) = \angle(Tj\omega \pm 1)$

At low frequency $= \angle \pm 1 =$

At high frequency $= \angle Tj\omega =$

At frequency $(T\omega \approx 1) = \angle Tj\omega \pm 1 \approx$

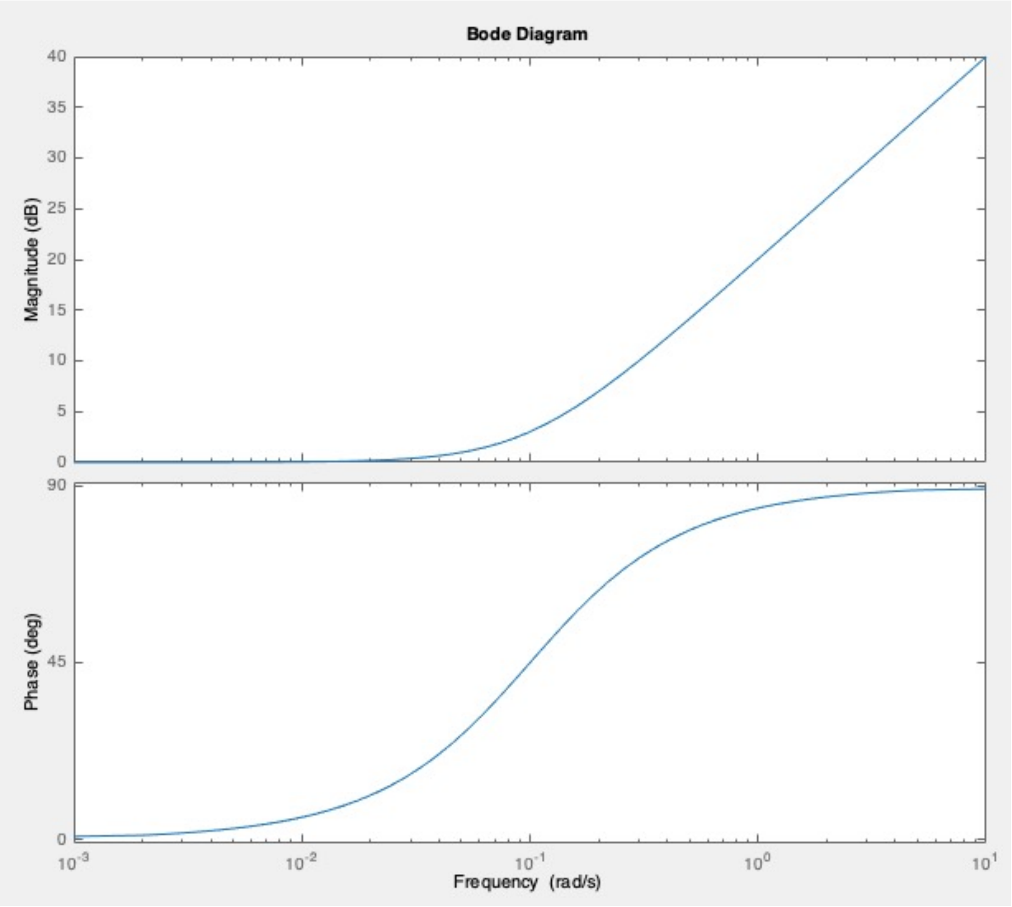
Two lines are connected by a third line at $\omega = \frac{1}{10T}$ and $\omega = \frac{10}{T}$

Example

$$G(s) = 10s + 1$$

$$20\log_{10}|G(jw)| = 20\log_{10}|j10\omega + 1|$$

$$\angle G(jw) = \angle(j10\omega + 1)$$

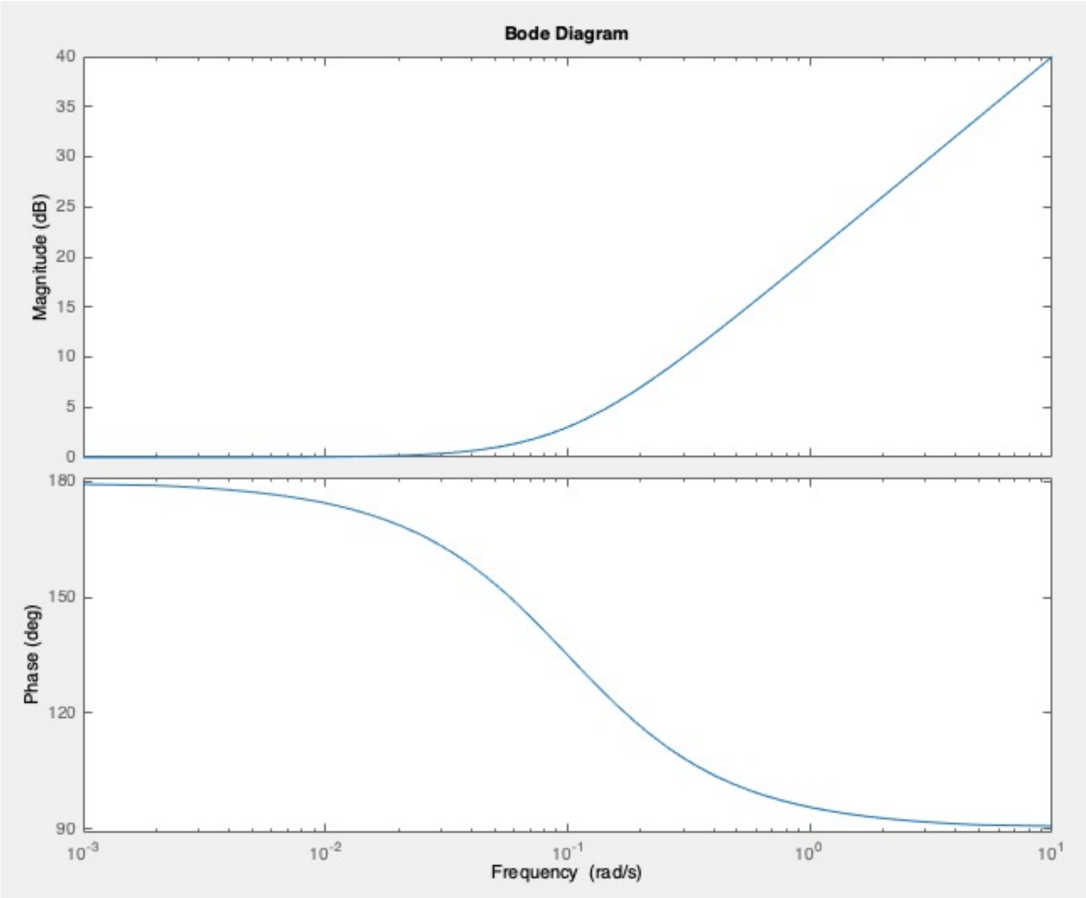


Example

$$G(s) = 10s - 1$$

$$20\log_{10}|G(jw)| = 20\log_{10}|j10\omega - 1|$$

$$\angle G(jw) = \angle(j10\omega - 1)$$



Example

$G(s) = \frac{1}{Ts \pm 1}$ (T positive)

$20\log_{10}|G(jw)| = 20\log_{10}|\frac{1}{Tj\omega \pm 1}|$

At low frequency $= 20\log_{10}|\frac{1}{\pm 1}| =$

At high frequency $= 20\log_{10}|\frac{1}{Tj\omega}| =$

Two lines meet at $\omega = \frac{1}{T}$ (break point)

Example

$G(s) = \frac{1}{Ts \pm 1}$ (T positive)

$\angle G(jw) = \angle \frac{1}{Tj\omega \pm 1}$

At low frequency $= \angle \frac{1}{\pm 1} =$

At high frequency $= \angle \frac{1}{Tj\omega} =$

At frequency $(T\omega \approx 1) = \angle \frac{1}{Tj\omega \pm 1} \approx$

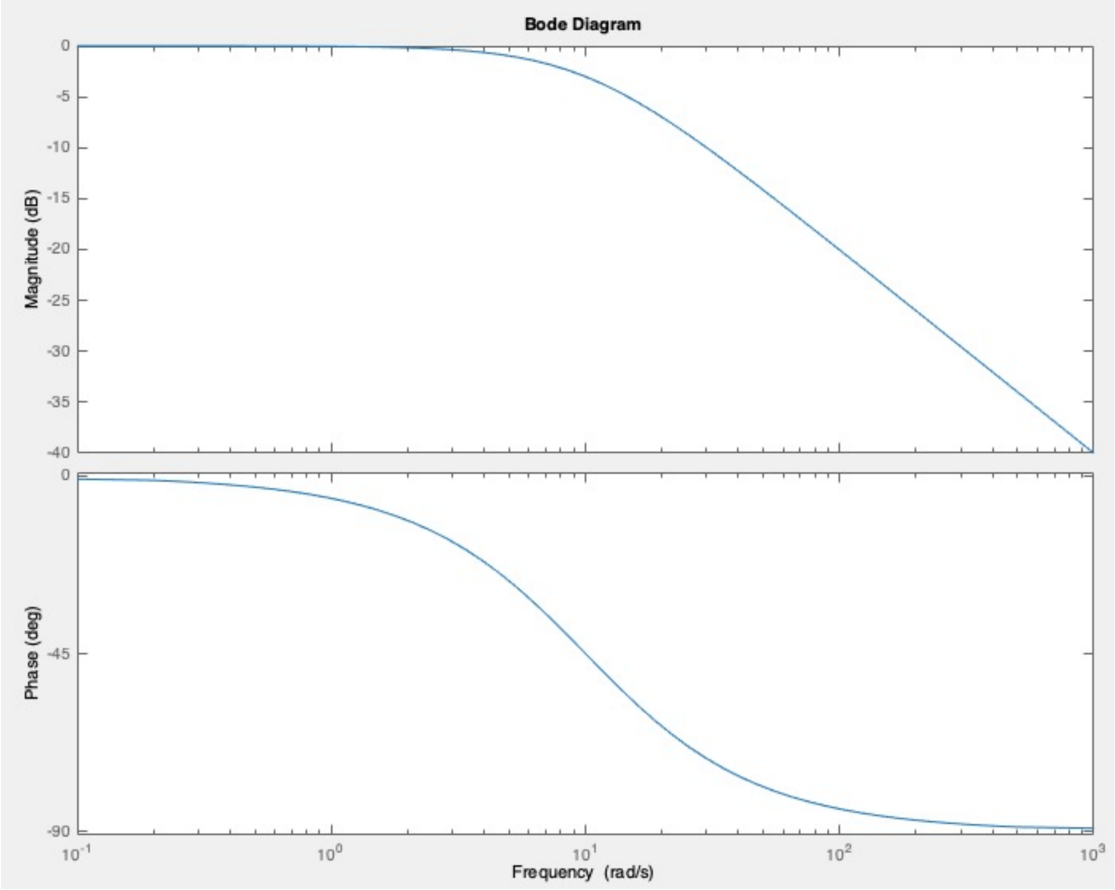
Two lines are connected by a third line at $\omega = \frac{1}{10T}$ and $\omega = \frac{10}{T}$

Example

$$G(s) = \frac{1}{0.1s+1}$$

$$20\log_{10}|G(jw)| = 20\log_{10}|\frac{1}{0.1j\omega+1}|$$

$$\angle G(jw) = \angle\frac{1}{0.1j\omega+1}$$

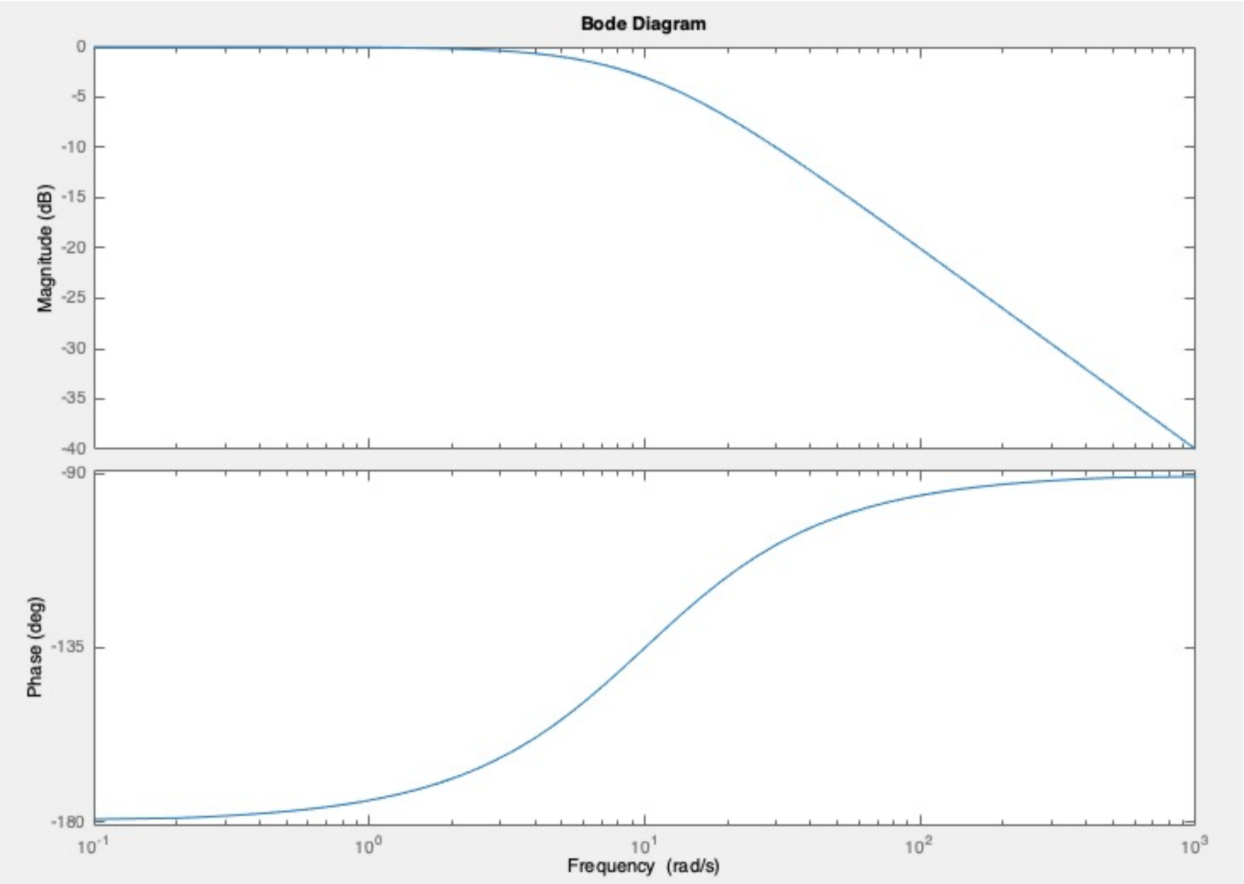


Example

$$G(s) = \frac{1}{0.1s-1}$$

$$20\log_{10}|G(jw)| = 20\log_{10}|\frac{1}{0.1j\omega-1}|$$

$$\angle G(jw) = \angle\frac{1}{0.1j\omega-1}$$



Example

$$G(s) = \frac{100}{s(s+10)}$$

$$20\log_{10}|G(jw)| =$$

$$\angle G(jw) =$$

Example

$$G(s) = \frac{100}{s(s+10)} = \frac{10}{s(0.1s+1)}$$

$$20\log_{10}|G(jw)| = 20\log_{10}|\frac{10}{jw}| + 20\log_{10}|\frac{1}{0.1jw+1}|$$

$$\angle G(jw) = \angle\frac{10}{jw} + \angle\frac{1}{0.1jw+1}$$