Automatica 119 (2020) 109094

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Distributed output regulation of heterogeneous uncertain linear agents ☆

Satoshi Kawamura [a], Kai Cai [a,*], Masako Kishida [b]

[a] Osaka City University, Japan
[b] National Institute of Informatics, Japan

ARTICLE INFO

Article history:
Received 26 September 2018
Received in revised form 18 December 2019
Accepted 28 May 2020
Available online 20 June 2020

ABSTRACT

We study a multi-agent output regulation problem, where only a few agents have access to the exosystem's dynamics. We propose a fully distributed controller that solves the problem for linear, heterogeneous, non-minimum-phase, and uncertain agent dynamics as well as time-varying directed networks. The distributed controller consists of two parts: (1) an *exosystem generator* that locally estimates the exosystem dynamics, and (2) a *dynamic compensator* that, by locally approaching the internal model of the exosystem, achieves perfect output regulation. Compared to the existing results in the literature, our solution is the first to solve the distributed output regulation problem where not all agents have access to information of the exosystem and the agent dynamics are uncertain and non-minimum-phase.

1. Introduction

In cooperative control of multi-agent systems, the *distributed output regulation problem* with general linear and heterogeneous agent dynamics has received much recent attention (e.g. Cai, Lewis, Hu, & Huang, 2017; Liu & Huang, 2015; Su, Hong, & Huang, 2013; Su & Huang, 2012; Wang, Hong, Huang, & Jiang, 2010; Yan & Huang, 2017). In this problem, a network of agents each tries to match its output with a reference signal, under the constraint that only a few agents can directly measure the reference. The reference signal itself is generated by an external dynamic system, called "exosystem". The distributed output regulation problem not only subsumes earlier problems such as (leader-following) consensus and synchronization, but also addresses issues of disturbance rejection and robustness to parameter uncertainty. Also see e.g. Liu and Huang (2017, 2018) and Su and Huang (2013) for further extensions that deal with nonlinear agent dynamics.

Output regulation has a well-studied centralized version: A single plant tries to match its output with a reference signal (while maintaining the plant's internal stability) (Francis, 1977; Francis & Wonham, 1976). In the absence of system parameter uncertainty, the solution of the "regulator equations" provides a solution to output regulation (Francis, 1977). When system parameters are subject to uncertainty, however, a dynamic compensator must be used embedding q-copy of the exosystem, where q is the number of (independent) output variables to be regulated. The latter is well-known as the *internal model principle* (Francis & Wonham, 1976). These methods for solving the centralized output regulation problem, however, cannot be applied directly to the distributed version, inasmuch as not all agents have direct access to the reference signal or the exosystem dynamics.

The distributed output regulation of networks of heterogeneous linear agents is studied in Su and Huang (2012). The proposed distributed controller consists of two parts: an exosystem generator and a controller based on regulator equation solutions. Specifically, the exosystem generator of each agent aims to (asymptotically) synchronize with the exosystem using consensus protocols, thereby creating a local estimate of the exosystem. Meanwhile each agent independently tracks the signal of its local generator, by applying standard centralized methods (in Su & Huang, 2012 regulator equation solutions). This approach effectively separates the controller synthesis into two parts — distributed exosystem generators by network consensus and local output regulation by regulator equation solution.

One important limitation, however, of the above approach is: in both the exosystem generator design and the regulator

☆ This work was supported in part by the open collaborative research program at National Institute of Informatics (NII) Japan (FY2018) and the Research and Development of Innovative Network Technologies to Create the Future of National Institute of Information and Communications Technology (NICT) of Japan. The material in this paper was partially presented at the 58th IEEE Conference on Decision and Control, December 11–13, 2019, Nice, France. This paper was recommended for publication in revised form by Associate Editor Claudio De Persis under the direction of Editor Christos G. Cassandras.

* Corresponding author.
E-mail addresses: kawamura@c.info.eng.osaka-cu.ac.jp (S. Kawamura), kai.cai@eng.osaka-cu.ac.jp (K. Cai), kishida@nii.ac.jp (M. Kishida).

https://doi.org/10.1016/j.automatica.2020.109094
0005-1098/

equation solution, it is assumed that each agent uses exactly the same dynamic model as that of the exosystem. This assumption may be unreasonable in the distributed network setting, because those agents that cannot directly measure the reference are unlikely to know the dynamic model of the exosystem. To deal with this challenge, Cai et al. (2017) propose (for static networks) an "adpative" exosystem generator and an adaptive solution to the regulator equations. In essence, each agent runs an additional consensus algorithm to update their "local estimates" of the exosystem dynamics.

All the regulator-equation based solutions above fall short in addressing the issue of system parameter uncertainty. In practice one may not have precise knowledge of some entries of the system matrices, or the values of some parameters may drift over time. The distributed output regulation problem considering parameter uncertainty is studied in Su et al. (2013) and Wang et al. (2010). The proposed controller is based on the internal model principle, but does not employ the two-part structure mentioned above. It appears to be for this reason that restrictive conditions (acyclic graph or homogeneous nominal agent dynamics) have to be imposed in order to ensure solving output regulation. Moreover, it is also assumed in Su et al. (2013) and Wang et al. (2010) that each agent knows the exact model of the exosystem dynamics.

In this paper, we provide a new solution to the distributed output regulation problem of heterogeneous linear agents, where not all agents have direct access to the dynamic model of the exosystem and the agent dynamics are subject to parameter uncertainty. In this setting, to our best knowledge, no solution exists in the literature. In particular, we propose to use the two-part structure of the distributed controller in the following manner: The first part is an exosystem generator that works over time-varying networks (Liu & Huang, 2017), and the second part is a dynamic compensator embedding an internal model of the exosystem that addresses parameter uncertainty. The challenge here is, in the design of the dynamic compensator, those agents that cannot directly measure the exosystem have *no* knowledge of the internal model of the exosystem; on the other hand, we know from Francis and Wonham (1976) that a precise internal model is necessary to achieve perfect regulation with uncertain parameters. To deal with this challenge, we propose a consensus-based local internal model for each agent to estimate the internal model of the exosystem. For this time-varying local internal model, we moreover design novel strategies for its eigenvalues to avoid certain transmission zeros of the agents' dynamics (generally non-minimum-phase) in order to guarantee the existence of a dynamic compensator for all time.

The contributions of this paper are twofold. First, the proposed internal-model based distributed controller is the first solution to the multi-agent output regulation problem where not all agents have direct access to the internal model (q-copy) of the exosystem and the agent dynamics are uncertain and non-minimum-phase. Concretely, the proposed distributed controller provably solves the multi-agent output regulation problem in which the following constraints/conditions simultaneously hold: (a) Only a (proper) subset of agents have access to the dynamic model of the exosystem. This is not considered in Su et al. (2013) and Wang et al. (2010). (b) Parameter uncertainty of agent dynamics. This is not addressed in Cai et al. (2017) and Su and Huang (2012). (c) Non-minimum phase agent dynamics. This is not dealt with in Liu and Huang (2015). (d) Time-varying directed networks. This is not addressed in Cai et al. (2017), Su and Huang (2013) and Yan and Huang (2017). (e) Heterogeneous agent dynamics. This is not dealt with in Yan and Huang (2017). As a second contribution, the core of our solution is the time-varying local internal model (q-copy), updated in the network setting, which is in itself new in the literature of the internal model principle (cf. Francis, 1977; Francis & Wonham, 1976; Huang, 2004) and generalizes the (static, centralized) internal model to the dynamic, distributed one.

2. Preliminaries

In this paper, we will use the following notation. Let $\mathbf{1}_n := [1 \cdots 1]^\top \in \mathbb{R}^n$, and I_n be the $n \times n$ identity matrix. For a complex number $c \in \mathbb{C}$, denote its complex conjugate by c^*. Write $\mathbb{C}_+$ for the closed right half (complex) plane; $\sigma(A)$ for the set of all eigenvalues of A. We say that a (square) matrix is stable if the real parts of all its eigenvalues are negative.

2.1. Agents and exosystem

We consider a network of N agents that are linear, time-invariant, and finite-dimensional. The dynamics of each agent $i(= 1, \ldots, N)$ is given by

$$\dot{x}_i = A_i x_i + B_i u_i + P_i w_0 \tag{1}$$

$$z_i = C_i x_i + D_i u_i + Q_i w_0 \tag{2}$$

where $x_i \in \mathbb{R}^{n_i}$ is the state vector, $u_i \in \mathbb{R}^{m_i}$ the control input, $z_i \in \mathbb{R}^{q_i}$ the output to be regulated, and $w_0 \in \mathbb{R}^r$ the *exogenous signal* generated by the *exosystem*

$$\dot{w}_0 = S_0 w_0. \tag{3}$$

Here $A_i, B_i, C_i, D_i, P_i, Q_i$ and S_0 are real matrices of appropriate sizes. The signal w_0 represents reference to be tracked and/or disturbance to be rejected: $P_i w_0$ in (1) represents disturbance acting on agent i's dynamics and $Q_i w_0$ in (2) represents reference signals to be tracked.

Assumption 1. The exosystem's w_0 and S_0 can be accessed only by a proper subset of the N agents. Those agents that have access to w_0, S_0 cannot distinguish w_0, S_0 of the exosystem respectively from w_i, S_i of other (neighbor) agents.

Note that the agents are generally heterogeneous: Each of the matrices A_i, B_i, C_i, D_i, P_i and Q_i may have different dimensions and entries. Furthermore, we consider that the matrices may have uncertainty; namely

$$A_i = A_{i0} + \Delta A_i,\ \ B_i = B_{i0} + \Delta B_i,\ \ C_i = C_{i0} + \Delta C_i,$$
$$D_i = D_{i0} + \Delta D_i,\ \ P_i = P_{i0} + \Delta P_i,\ \ Q_i = Q_{i0} + \Delta Q_i \tag{4}$$

where $A_{i0}, B_{i0}, C_{i0}, D_{i0}, P_{i0}, Q_{i0}$ are the nominal parts of agent i and $\Delta A_i, \Delta B_i, \Delta C_i, \Delta D_i, \Delta P_i, \Delta Q_i$ are the uncertain parts. These uncertainty parts may represent measurement errors in the actual determination of the physical parameters, or the reality that these parameters may change with time due to wear and aging.

2.2. Communication digraphs

Given a multi-agent system with $N(\geq 1)$ agents and an exosystem, we represent the time-varying interconnection among the agents and the exosystem by a digraph $\hat{\mathcal{G}}(t) = (\hat{\mathcal{V}}, \hat{\mathcal{E}}(t))$, where $\hat{\mathcal{V}} = \mathcal{V} \cup \{0\}$, $\mathcal{V} = \{1, \ldots, N\}$, is the node set, and $\hat{\mathcal{E}}(t) \subseteq \hat{\mathcal{V}} \times \hat{\mathcal{V}}$ is the edge set. The node i ($\in \{1, \ldots, N\}$) represents the ith agent, and the node 0 the exosystem. Moreover, $\hat{\mathcal{V}}$ is the node set including the exosystem and $\mathcal{V}$ is the node set except for the exosystem. The ith node receives information from the jth (neighbor) node at time t if and only if $(j, i) \in \hat{\mathcal{E}}(t)$. We consider the digraph $\hat{\mathcal{G}}(t)$ that does not contain selfloop edges at all times, i.e. $(i, i) \notin \hat{\mathcal{E}}(t)$ for all $i \in \hat{\mathcal{V}}$ and $t \geq 0$. Only those nodes $i \in \mathcal{V}$ such that $(0, i) \in \hat{\mathcal{E}}(t)$ can receive information from the exosystem 0 (i.e. w_0, S_0) at time t. The *union digraph* for a time interval $[t_1, t_2]$ ($0 \leq t_1 \leq t_2$) is defined as $\hat{\mathcal{G}}([t_1, t_2]) := (\hat{\mathcal{V}}, \cup_{t \in [t_1, t_2]} \hat{\mathcal{E}}(t))$.

Definition 1. The digraph $\hat{\mathcal{G}}(t)$ *uniformly contains a spanning tree* if there is $T > 0$ such that for every $t \geq 0$ the union digraph $\hat{\mathcal{G}}([t, t+T])$ contains a spanning tree.

We define the communication weight $a_{ij}(t)$ by $a_{ij}(t) \geq \epsilon$ (where ϵ is a positive constant) if $(j, i) \in \hat{\mathcal{E}}(t)$, and $a_{ij}(t) = 0$ if $(j, i) \notin \hat{\mathcal{E}}(t)$. (We assume that $a_{ij}(t)$ is piecewise continuous and bounded for all $t \geq 0$, a technical assumption needed for proving Theorem 1). Note that the exosystem does not receive information from any agents, and thus $a_{0j}(t) = 0$ for all $j \in \mathcal{V}$, $t \geq 0$.

For time $t \geq 0$ and digraph $\hat{\mathcal{G}}(t)$, the graph Laplacian $L(t) = [l_{ij}(t)] \in \mathbb{R}^{(N+1)\times(N+1)}$ is defined as

$$l_{ij}(t) := \begin{cases} \sum_{j=0}^{N} a_{ij}(t), & i = j \\ -a_{ij}(t), & i \neq j \end{cases} \qquad (i, j \in \{0, \ldots, N\})$$

2.3. Problem statement

We represent by $\hat{\mathcal{G}}(t) = (\hat{\mathcal{V}}, \hat{\mathcal{E}}(t))$ the time-varying interconnection among the N agents and the exosystem as in the preceding subsection. In particular, at any time only a proper subset of agents (possibly different across time) can receive information from the exosystem. This makes the current problem different from the traditional, centralized output regulation problem (Francis, 1977; Francis & Wonham, 1976; Huang, 2004). Even if an agent receives information from the exosystem at some time, the agent does not know whether the information is from the exosystem or from another (neighbor) agent. Namely we consider that the agents do not have the numbering information including the exosystem (numbered 0).

Problem 1 (*Distributed Output Regulation*)**.** Given a network of agents (1), (2), (4) and an exosystem (3) with interconnection represented by $\hat{\mathcal{G}}(t)$ and with Assumption 1, design for each agent $i \in \mathcal{V}$ a distributed controller such that $\lim_{t\to\infty} z_i(t) = 0$, for all $x_i(0) \in \mathbb{R}^{n_i}$, $w_0(0) \in \mathbb{R}^r$.

In the next section we solve Problem 1 by designing an internal-model based distributed controller.

3. Structure of distributed controller

At the outset we make the following (standard) assumptions.

Assumption 2. The digraph $\hat{\mathcal{G}}(t)$ uniformly contains a spanning tree and its root is node 0 (the exosystem).

Assumption 3. For each agent $i \in \mathcal{V}$, (A_{i0}, B_{i0}) is stabilizable.

Assumption 4. For each agent $i \in \mathcal{V}$, (C_{i0}, A_{i0}) is detectable.

Assumption 5. For each agent $i \in \mathcal{V}$ and for every eigenvalue λ of S_0,

$$\text{rank} \begin{bmatrix} A_{i0} - \lambda I_{n_i} & B_{i0} \\ C_{i0} & D_{i0} \end{bmatrix} = n_i + q_i. \qquad (5)$$

Assumption 6. The real parts of all eigenvalues of S_0 are zeros.

Remark 1. Assumptions 2 and 3–5 are necessary conditions for consensus over time-varying networks (Ren & Beard, 2008) and for output regulation (Francis & Wonham, 1976), respectively. Only Assumption 6 is a sufficient condition for (centralized) output regulation, but is commonly made for distributed output regulation (e.g. Cai et al., 2017; Liu & Huang, 2015) such that the exogenous signal does not diverge exponentially fast.

Remark 2. By Huang (2004), Assumption 5 means that the *transmission zeros* of agent i's nominal dynamics are disjoint from all eigenvalues of S_0, and also implies that the number of outputs is no more than that of inputs, i.e. $m_i \geq q_i$. A transmission zero $\zeta \in \mathbb{C}$ of agent i's nominal dynamics is such that

$$\text{rank} \begin{bmatrix} A_{i0} - \zeta I_{n_i} & B_{i0} \\ C_{i0} & D_{i0} \end{bmatrix} < n_i + q_i.$$

Because not all agents can access the exosystem (i.e. w_0 cannot be accessed by all agents), agent i cannot measure the output z_i in (2) directly. Replacing w_0 in (2) by an *estimated* exogenous signal $w_i \in \mathbb{R}^r$, we consider the situation where each agent i can measure the following *estimated* output

$$e_i = C_i x_i + D_i u_i + Q_i w_i \in \mathbb{R}^{q_i} \qquad (6)$$

In order to solve Problem 1, we present a controller that consists of two parts: (1) distributed *exosystem generator* and (2) distributed *dynamic compensator*.

3.1. Distributed exosystem generator

For each agent $i \in \mathcal{V}$, let $S_i(t) \in \mathbb{R}^{r\times r}$ be the local estimate of S_0 and consider

$$\dot{S}_i(t) = \sum_{j=0}^{N} a_{ij}(t) \left(S_j(t) - S_i(t)\right), \qquad (7)$$

$$\dot{w}_i(t) = S_i(t) w_i(t) + \sum_{j=0}^{N} a_{ij}(t) \left(w_j(t) - w_i(t)\right). \qquad (8)$$

Eqs. (7) and (8) have also been used in Liu and Huang (2017) for the "adaptive distributed observer", and first proposed in Cai et al. (2017) for time-invariant networks. This protocol is used to approximate the exosystem's dynamics for each agent $i \in \mathcal{V}$ using only local information from neighbors. Thus we call (7) and (8) the *distributed exosystem generator*.

Lemma 1. *Consider the distributed exosystem generator (7) and (8). If Assumption 2 holds, then*

$$\lim_{t\to\infty} (S_i(t) - S_0) = 0, \quad \lim_{t\to\infty} (w_i(t) - w_0) = 0$$

for all $S_i(0) \in \mathbb{R}^{r\times r}$, $w_i(0) \in \mathbb{R}^r$ and all $i \in \mathcal{V}$.

A proof of Lemma 1 can be found in Kawamura, Cai, and Kishida (2018) or Liu and Huang (2017).

3.2. Distributed dynamic compensator

We consider the following *dynamic compensator*

$$\dot{\xi}_i = E_i(t)\xi_i + F_i(t) e_i$$
$$u_i = K_i(t)\xi_i \qquad (9)$$

where ξ_i is the state of the dynamic compensator and e_i is defined in (6). In order to specify the matrices $E_i(t)$, $F_i(t)$, $K_i(t)$ in (9), we extend the centralized internal model control design in Huang (2004, Section 1.3) to the distributed multi-agent setting. The key strategy is for each agent to locally estimate the (distinct) eigenvalues of the exosystem's matrix S_0.

Let $\lambda_{0,1}, \ldots, \lambda_{0,k}$, $k \leq r$ be the roots of the minimal polynomial of S_0. Note that $\{\lambda_{0,1}, \ldots, \lambda_{0,k}\} \subseteq \sigma(S_0)$. Then we define $\lambda_0 := [\lambda_{0,1} \cdots \lambda_{0,k}]^\top$. Let $c_{0,d}(\lambda_0)$, $d = 1, \ldots, k$ be the coefficients of the polynomial satisfying

$$s^k + c_{0,1}(\lambda_0)s^{k-1} + \cdots + c_{0,k-1}(\lambda_0)s + c_{0,k}(\lambda_0) = (s - \lambda_{0,1}) \cdots (s - \lambda_{0,k}). \qquad (10)$$

For each agent $i \in \mathcal{V}$, let $\lambda_i(t) := [\lambda_{i,1}(t)\cdots\lambda_{i,k}(t)]^\top$ be a local estimate of λ_0, and $c_{i,d}(\lambda_i), d = 1,\ldots,k$, the estimated coefficients generated by $\lambda_i(t)$ that satisfy

$$\begin{aligned} s^k + c_{i,1}(\lambda_i)s^{k-1} + \cdots + c_{i,k-1}(\lambda_i)s + c_{i,k}(\lambda_i) \\ = (s-\lambda_{i,1}(t))\cdots(s-\lambda_{i,k}(t)). \end{aligned} \tag{11}$$

To make the local estimate λ_i converge to λ_0, a straightforward idea is to employ the following consensus algorithm:

$$\dot{\lambda}_i(t) = \sum_{j=0}^{N} a_{ij}(t)\left(\lambda_j(t) - \lambda_i(t)\right), \ \ \lambda_i(0) \in \mathrm{j}\mathbb{R}^k. \tag{12}$$

It follows from Assumption 2 that $\lambda_i(t) \to \lambda_0$ as $t \to \infty$. As a result, the coefficient $c_{i,d}(\lambda_i) \to c_{0,d}(\lambda_0)$ as $t \to \infty$ for each $d = 1,\ldots,k$. Note that by Assumption 6 the entries of λ_0 are purely imaginary, and hence we only need to consider the initial condition $\lambda_i(0) \in \mathrm{j}\mathbb{R}^k$ (thus $\lambda_i(t) \in \mathrm{j}\mathbb{R}^k$ for all $t \geq 0$).

Since we consider that the agents' dynamics have uncertainty, the regulator equation approach (e.g. Cai et al., 2017) does not work. Thus for the robust output regulation problem, we consider the q_i-copy *internal model* as in Huang (2004, Section 1.3). Let $G_i(\lambda_i) := I_{q_i} \otimes G_i'(\lambda_i),\ H_i := I_{q_i} \otimes H_i'$ be the q_i-copy internal model ($\otimes$ denotes Kronecker product), where

$$\begin{aligned} G_i'(\lambda_i) &:= \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -c_{i,k}(\lambda_i) & -c_{i,k-1}(\lambda_i) & \cdots & -c_{i,1}(\lambda_i) \end{bmatrix}, \\ H_i' &:= \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}. \end{aligned} \tag{13}$$

We state the following lemma using the above matrices.

Lemma 2. *Assume Assumption 3 holds. Let $t \geq 0$ and $\lambda_i(t) = [\lambda_{i,1}(t)\cdots\lambda_{i,k}(t)]^\top$. If for every $d \in \{1,\ldots,k\}$,*

$$\mathrm{rank}\begin{bmatrix} A_{i0} - \lambda_{i,d}(t)I_{n_i} & B_{i0} \\ C_{i0} & D_{i0} \end{bmatrix} = n_i + q_i \tag{14}$$

then the following pair of matrices is stabilizable:

$$\left(\begin{bmatrix} A_{i0} & 0 \\ H_iC_{i0} & G_i(\lambda_i(t)) \end{bmatrix}, \begin{bmatrix} B_{i0} \\ H_iD_{i0} \end{bmatrix}\right).$$

Lemma 2 asserts that for each fixed timed t, the above pair of matrices is stabilizable if (14) holds. For a proof refer to Kawamura et al. (2018). This sufficient condition (14) means that every $\lambda_{i,d}$ ($d \in \{1,\ldots,k\}$) does not coincide with any transmission zeros of agent i's nomial dynamics. Updated by (12), $\lambda_{i,d}$ is time-varying and $\lambda_{i,d}(t) \in \mathrm{j}\mathbb{R}$ for all $t \geq 0$. If agent i's nominal dynamics has no purely imaginary transmission zero, then (14) holds for all time t. In case agent i's nominal dynamics has purely imaginary transmission zeros, however, it is possible that (14) is violated. This is because in (12) the (vector) $\lambda_i(t) = [\lambda_{i,1}(t)\cdots\lambda_{i,k}(t)]^\top$ is updated continuously, and entries $\lambda_{i,d}(t) \in \mathrm{j}\mathbb{R}$ ($d \in \{1,\ldots,k\}$) may coincide with those purely imaginary transmission zeros at some time t. Thus for this case, the simple consensus algorithm (12) does not work.

To address this issue, we design a new strategy to update the (vector) $\lambda_i(t) = [\lambda_{i,1}(t)\cdots\lambda_{i,k}(t)]^\top$ such that for all $d \in \{1,\ldots,k\}$ (i) $\lambda_{i,d}(t)$ converges to $\lambda_{0,d}$, and (ii) $\lambda_{i,d}(t)$ avoids the transmission zeros of agent i's nomial dynamics for all $t \geq 0$. Thereby, it is ensured that the condition (14) holds for all time t.

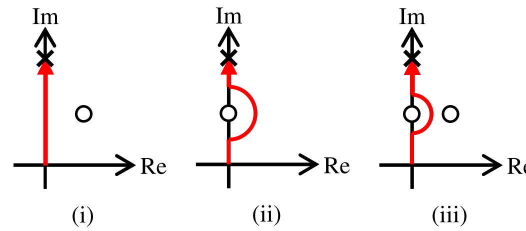


Fig. 1. The trajectory of estimated eigenvalues $\lambda_{i,d}$; the circles represent the transmission zeros of agent i, the crosses represent the eigenvalues of S_0, and the arrows represent the trajectories of $\lambda_{i,d}(t)$.

Fig. 1 shows examples of the trajectory of $\lambda_{i,d}$ ($d \in \{1,\ldots,k\}$). The circles and the crosses represent respectively the transmission zeros of the agent i's nomial dynamics and the eigenvalue $\lambda_{0,d}$ of S_0. The initial value $\lambda_{i,d}(0)$ is in $\mathrm{j}\mathbb{R}$ and $\lambda_{i,d}(t)$ moves toward $\lambda_{0,d}$. We divide the arrangement of transmission zeros into three cases:

- (i) If there is no purely imaginary transmission zero of agent i's nomial dynamics (see Fig. 1(i)), then $\lambda_{i,d}(t)$ may simply move along the imaginary axis toward $\lambda_{0,d}$.
- (ii) If there is a purely imaginary transmission zero of agent i's nomial dynamics, and $\lambda_{i,d}(t)$ moves close to it (see Fig. 1(ii)), then $\lambda_{i,d}(t)$ should move in a semicircle dented to the right around the transmission zero. By moving to the right, $\lambda_{i,d}(t)$ is always in $\mathbb{C}_+$.
- (iii) If there is a purely imaginary transmission zero of agent i's nomial dynamics, and there are also other transmission zeros on the open right-half-plane (see Fig. 1(iii)), the radius of semicircle should be smaller than (e.g. half of) the distance between these transmission zeros.

To formalize the above idea, we define several quantities. Let

$$\varPi_i := \left\{ s \in \mathbb{C}_+ \middle| \mathrm{rank}\begin{bmatrix} A_{i0} - sI_{n_i} & B_{i0} \\ C_{i0} & D_{i0} \end{bmatrix} < n_i + q_i \right\} \tag{15}$$

be the set of closed right-half-plane transmission zeros of agent i's nominal dynamics ($i \in \mathcal{V}$) and

$$\tilde{\varPi}_i := \{ s \in \varPi_i \mid \mathrm{Re}(s) = 0 \} \tag{16}$$

the subset of purely imaginary transmission zeros. We do not need to avoid open left-half-plane transmission zeros because $\lambda_{i,d}(t) \in \mathbb{C}_+$ for all t. Note that Assumption 5 and $\varPi_i \cap \sigma(S_0) = \emptyset$ are equivalent, and there is no purely imaginary transmission zeros of agent i's nominal dynamics if and only if $\tilde{\varPi}_i = \emptyset$.

Further we define the following distance function. For two sets $C_1, C_2 \subseteq \mathbb{C}$ of finite number of complex numbers, define the *distance* between C_1 and C_2 by

$$\mathrm{dist}(C_1, C_2) := \min\{|c_1 - c_2| \mid c_1 \in C_1, c_2 \in C_2\}.$$

Then define the radius $\rho_i \geq 0$ of the semicircle shown in Fig. 1 as

$$\rho_i := \begin{cases} 0, & \tilde{\varPi}_i = \emptyset \\ \frac{1}{2}\mathrm{dist}(\sigma(S_0), \tilde{\varPi}_i), & \varPi_i \setminus \tilde{\varPi}_i = \emptyset \\ \frac{1}{2}\min\{\mathrm{dist}(\sigma(S_0), \tilde{\varPi}_i), \mathrm{dist}(\varPi_i \setminus \tilde{\varPi}_i, \tilde{\varPi}_i)\}, & \text{otherwise}. \end{cases} \tag{17}$$

This ρ_i has three cases:

- (i) If there is no purely imaginary transmission zeros of agent i's nominal dynamics, i.e. $\tilde{\varPi}_i = \emptyset$, then the radius is zero.

(ii) If there are only purely imaginary transmission zeros of agent i's nominal dynamics, i.e. $\Pi_i \setminus \tilde{\Pi}_i = \emptyset$, then the radius is half of the distance between $\sigma(S_0)$ and $\tilde{\Pi}_i$.
(iii) If there are transmission zeros of agent i's nominal dynamics on both purely imaginary axis and right-half-plane, then the radius is half of the smaller of the distance between $\sigma(S_0)$ and $\tilde{\Pi}_i$ and that between $\Pi_i \setminus \tilde{\Pi}_i$ and $\tilde{\Pi}_i$.

In the definition of ρ_i, we consider the coefficient 1/2 for simplicity, but we can choose any coefficient from the open interval (0, 1). By using this ρ_i, we ensure the radius of the semicircle for the three cases as illustrated in Fig. 1.

Then we consider the following updates for all $i \in \hat{\mathcal{V}}$ and $t \geq 0$:

$$\lambda_i(t) = \alpha_i(t) + \mathrm{j}\beta_i(t) \in \mathbb{C}^k \tag{18}$$

$$\dot{\beta}_i(t) = \sum_{j=0}^{N} a_{ij}(t)\left(\beta_j(t) - \beta_i(t)\right), \beta_i(0) \in \mathbb{R}^k \tag{19}$$

$$\alpha_{i,d}(t) = \begin{cases} 0, & \gamma_{i,d}(t) \geq \rho_i \text{ or } i = 0 \\ \sqrt{\rho_i^2 - \gamma_{i,d}^2(t)}, & \gamma_{i,d}(t) < \rho_i \text{ and } i \neq 0 \end{cases} \tag{20}$$

where $d = 1, \ldots, k$, $\alpha_i(t) = [\alpha_{i,1}(t) \cdots \alpha_{i,k}(t)]^\top$, $\beta_i(t) = [\beta_{i,1}(t) \cdots \beta_{i,k}(t)]^\top$, and

$$\gamma_{i,d}(t) := \begin{cases} 0, & \tilde{\Pi}_i = \emptyset \\ \operatorname{dist}\left(\{\mathrm{j}\beta_{i,d}(t)\}, \tilde{\Pi}_i\right), & \text{otherwise.} \end{cases}$$

Note that $\gamma_{i,d}$ means the distance between $\mathrm{j}\beta_{i,d}$ and its closest (purely imaginary) transmission zero, and $\alpha_{i,d}(t) \geq 0$ for all $t \geq 0$. Moreover from Assumption 2, $\dot{\beta}_0(t) = 0$, $\alpha_0(t) = 0$ for all $t \geq 0$. Finally, we update λ_i by (18), (19), (20) (instead of (12)). It follows immediately from these definitions the following result.

Lemma 3. *Consider (18), (19), (20). If Assumption 2 holds, then for every $i \in \mathcal{V}$ and $d = 1, \ldots, k$, $\lambda_{i,d}(t)$ converges to $\lambda_{0,d}$, $\lambda_{i,d}(t) \in \mathbb{C}_+$ for all $t \geq 0$ and avoids all transmission zeros of agent i's nominal dynamics.*

From Lemma 3 the condition (14) holds, and thus the pair of matrices in Lemma 2 is stabilizable for every time $t \geq 0$. Hence for each $t \geq 0$, we may synthesize $[K_{i1}(\lambda_i)\ K_{i2}(\lambda_i)]$ such that the matrix

$$\begin{bmatrix} A_{i0} & 0 \\ H_i C_{i0} & G_i(\lambda_i) \end{bmatrix} + \begin{bmatrix} B_{i0} \\ H_i D_{i0} \end{bmatrix} [K_{i1}(\lambda_i)\ K_{i2}(\lambda_i)] \tag{21}$$

is stable. In addition, we choose L_i such that the matrix $A_{i0} - L_i C_{i0}$ is stable under Assumption 4.

Now we are ready to present the matrices $E_i(t)$, $F_i(t)$ and $K_i(t)$ in the dynamic compensator (9):

$$E_i(\lambda_i) := \begin{bmatrix} A_{i0} - L_i C_{i0} & 0 \\ 0 & G_i(\lambda_i) \end{bmatrix} + \begin{bmatrix} B_{i0} - L_i D_{i0} \\ 0 \end{bmatrix} K_i(\lambda_i),$$

$$F_i := \begin{bmatrix} L_i \\ H_i \end{bmatrix}, \quad \text{and } K_i(\lambda_i) := [K_{i1}(\lambda_i)\ K_{i2}(\lambda_i)]. \tag{22}$$

Note that in (22), E_i and K_i are time-varying as λ_i is time-varying, while F_i is time-invariant; and by (18), (19), (20) it follows from Lemma 3 that $G_i(\lambda_i) \to G_i(\lambda_0)$, $K_i(\lambda_i) \to K_i(\lambda_0)$, $E_i(\lambda_i) \to E_i(\lambda_0)$. Using the distributed dynamic compensator (9), moreover, both the estimated output e_i and the output z_i can be regulated converge to 0.

Our main result is the following.

Theorem 1. *Consider the multi-agent system (1), (2), (4) and the exosystem (3), and suppose that Assumptions 1–6 hold. Then for each agent $i \in \mathcal{V}$, the distributed exosystem generator (7), (8) and the distributed dynamic compensator (9) with (22), (18), (19), (20) solve Problem 1.*

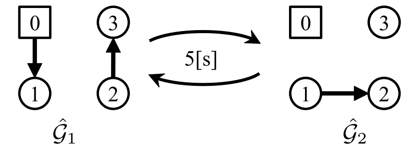


Fig. 2. Time-varying directed network.

A proof of Theorem 1 is in Kawamura et al. (2018). Several remarks on the result are in order.

Remark 3. Theorem 1 asserts that the proposed two-part distributed controller – the distributed exosystem generator (7), (8) and the distributed dynamic compensator (9) – provides the first solution to the multi-agent output regulation problem where not all agents have direct access to the exosystem's internal model and the agent dynamics are uncertain and non-minimum-phase. The key of our solution is the time-varying q_i-copy internal model, updated locally based only on information received from neighbors, which eventually converges to the exact internal model of the exosystem.

Remark 4. When there is only one agent (i.e. $N = 1$), the problem is specialized to the centralized output regulation, and Theorem 1 is thus an extension of the conventional results in Francis (1977), Francis and Wonham (1976) and Huang (2004).

Remark 5. In (19), we do not need to use all entries of $\beta_i = [\beta_{i,1}, \ldots, \beta_{i,k}]^\top$, because the eigenvalues of the real matrices S_i must be in conjugate pairs. Indeed, for all $i \in \hat{\mathcal{V}}$ we may write β_i in the following form

$$\beta_i(t) = \begin{cases} \left[\hat{\beta}_i(t)^\top \ -\hat{\beta}_i(t)^\top\right]^\top, & k \text{ is an even number} \\ \left[\hat{\beta}_i(t)^\top \ -\hat{\beta}_i(t)^\top\ 0\right]^\top, & k \text{ is an odd number} \end{cases} \tag{23}$$

where $\hat{\beta}_i \in \mathrm{j}\mathbb{R}^{\lfloor k/2 \rfloor}$. From this form, each agent can make their entire β_i after exchanging and updating only $\hat{\beta}_i$.

4. Simulation example

In this section, we illustrate the designed distributed controller by applying it to solve a distributed output regulation problem. Consider the time-varying network as displayed in Fig. 2. The network periodically switches between $\hat{\mathcal{G}}_1$ and $\hat{\mathcal{G}}_2$ every 5 s, and for simplicity we set the communication weight $a_{ij}(t) = 1$. Thus this network uniformly contains a spanning tree and its root is node 0, i.e. Assumption 2 holds.

The exosystem (node 0) is

$$\dot{w}_0(t) = S_0 w_0, \quad S_0 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Thus S_0 has two 0 eigenvalues, so Assumption 6 holds and β_0 in (19) is $\beta_0 = [0\ 0]^\top$. The agent $i(= 1, 2, 3)$ is

$$\dot{x}_i = (A_{i0} + \Delta A_i)x_i + (B_{i0} + \Delta B_i)u_i + (P_{i0} + \Delta P_i)w_0$$
$$z_i = (C_{i0} + \Delta C_i)x_i + (D_{i0} + \Delta D_i)u_i + (Q_{i0} + \Delta Q_i)w_0$$

where

$$A_{i0} = \begin{bmatrix} 0 & 1 \\ -i & 0 \end{bmatrix}, B_{i0} = \begin{bmatrix} 0 \\ (1/100)i^2 - (9/10)i + 1/4 \end{bmatrix},$$

$$P_{i0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C_{i0} = [1\ 0], D_{i0} = 1, Q_{i0} = [-1\ 0],$$

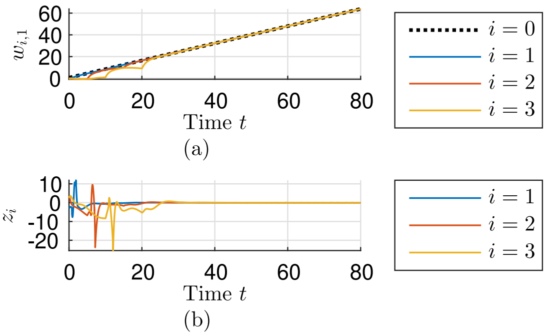


Fig. 3. Trajectories of $w_{i,1}$ ($i = 0, 1, 2, 3$) and z_i ($i = 1, 2, 3$).

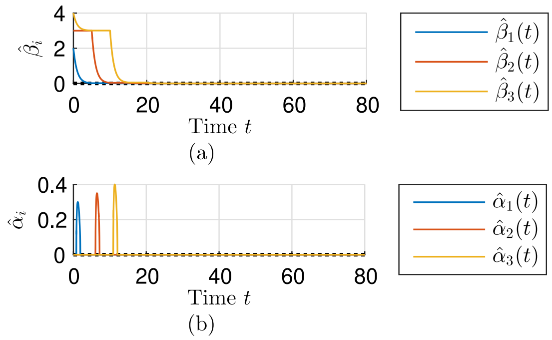


Fig. 4. Trajectories of real part $\hat{\alpha}_i$ and imaginary part $\hat{\beta}_i$ of local estimate λ_i, $i = 1, 2, 3$ ($\hat{\beta}_0(t) = 0, \hat{\alpha}_0(t) = 0$).

$$\Delta A_i = \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \end{bmatrix}, \Delta B_i = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \Delta P_i = \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \end{bmatrix},$$
$$\Delta C_i = [-0.05\ 0]\,, \Delta D_i = -0.05, \Delta Q_i = [0.1\ 0]\,.$$

It is checked that (A_{i0}, B_{i0}) and (C_{i0}, A_{i0}) are controllable and observable, respectively, and therefore stabilizable and detectable, respectively; thus Assumptions 3, 4 hold. The transmission zeros of the agents' nominal dynamics are

$$\Pi_i = \tilde{\Pi}_i = \{\pm(0.5 + 0.1i)\mathrm{j}\} \tag{24}$$

Thus all the transmission zeros are on the imaginary axis but nonzero, and as a result Assumption 5 holds.

We apply the distributed exosystem generator (7) and (8) with the initial conditions $w_i(0)$ ($i = 0, 1, 2, 3$), selected uniformly at random from $[-1, 1]$, and set

$$S_i(0) = \begin{bmatrix} 0 & 0.5 \\ -0.5 & 0 \end{bmatrix}.$$

We also apply the distributed dynamic compensator (9), (22), (18), (19), (20) with the elements of $x_i(0)$ and $\xi_i(0)$ selected uniformly at random from $[-1, 1]$, and set $\beta_i(0) = [i{+}1\ \ i{+}1]^\top$ for all $i = 1, 2, 3$. From (24), $\rho_i = (0.5 + 0.1i)/2$ and $\gamma_{i,d}(t)$ in (20) are $\gamma_{i,1}(t) = |0.5 + 0.1i - \beta_{i,1}(t)|$, $\gamma_{i,2}(t) = |-(0.5 + 0.1i) - \beta_{i,2}(t)|$.

The simulation result is displayed in Fig. 3. In Fig. 3(a), the dotted line represents the first element of exosystem's signal $w_{0,1}$ and others represent the estimated exogenous signals $w_{i,1}, i = 1, 2, 3$. Observe that all $w_{i,1}$ synchronize with $w_{0,1}$. Thus the distributed exosystem generators effectively create a local copy of the exosystem, despite that not all agents have direct access to the exosystem and the network is time-varying.

Fig. 3(b) shows the regulated output z_i of each agent (in this simulation, $z_i = x_{i,1} - w_{0,1}$). Observe that all z_i converge to 0. This demonstrates the effectiveness of the distributed dynamic compensators for achieving perfect regulation, despite parameter perturbation and imprecise internal model of the exosystem.

We finally examine the parameters in the distributed dynamic compensator. Consider $\hat{\beta}_i$ as in (23), and define $\hat{\alpha}_i$ to be the corresponding real part of local estimate λ_i based on (20). In this example, $\hat{\alpha}_i, \hat{\beta}_i \in \mathbb{R}$, $\alpha_i = [\hat{\alpha}_i\ \hat{\alpha}_i]^\top \in \mathbb{R}^2$ and $\beta_i = [\hat{\beta}_i\ -\hat{\beta}_i]^\top \in \mathbb{R}^2$. Fig. 4(a) and (b) show the trajectories of $\hat{\beta}_i$ and $\hat{\alpha}_i$, respectively. Each $\hat{\beta}_i$ converges to $\hat{\beta}_0$, and each $\hat{\alpha}_i$ becomes positive exactly when the distance between $\hat{\beta}_i$ and the closest transmission zeros to $\hat{\beta}_i$ (namely $\gamma_{i,d}(t)$) is less than ρ_i.

5. Conclusions

We have studied a multi-agent output regulation problem, where the linear agents are heterogeneous, non-minimum-phase, subject to parameter uncertainty, and the network is time-varying. We have solved the problem by proposing a distributed controller consisting of two parts – an exosystem generator that "learns" the dynamics of the exosystem and a dynamic compensator that "learns" the internal model. Our solution suggests a *distributed internal model principle*: converging internal models imply network output regulation.

References

Cai, H., Lewis, F. L., Hu, G., & Huang, J. (2017). The adaptive distributed observer approach to the cooperative output regulation of linear multi-agent systems. *Automatica*, *75*, 299–305.

Francis, B. A. (1977). The linear multivariable regulator problem. *SIAM Journal on Control and Optimization*, *15*(3), 486–505.

Francis, B. A., & Wonham, W. M. (1976). The internal model principle of control theory. *Automatica*, *12*(5), 457–465.

Huang, J. (2004). *Nonlinear output regulation: Theory and applications*. Society for Industrial and Applied Mathematics.

Kawamura, S., Cai, K., & Kishida, M. (2018). Distributed robust output regulation of heterogeneous uncertain linear agents by adaptive internal model principle. *Technical Report*, Available https://arxiv.org/abs/1803.00920.

Liu, W., & Huang, J. (2015). Cooperative robust output regulation of linear minimum-phase multi-agent systems under switching network. In *Proc. 10th Asian control conf* (pp. 1–5).

Liu, W., & Huang, J. (2017). Adaptive leader-following consensus for a class of higher-order nonlinear multi-agent systems with directed switching networks. *Automatica*, *79*, 84–92.

Liu, T., & Huang, J. (2018). Leader-following attitude consensus of multiple rigid body systems subject to jointly connected switching networks. *Automatica*, *92*, 63–71.

Ren, W., & Beard, R. (2008). *Distributed consensus in multi-vehicle cooperative control, Communications and control engineering series*. Springer.

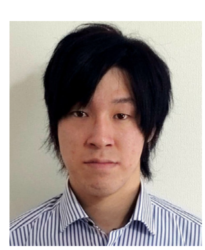
Su, Y., Hong, Y., & Huang, J. (2013). A general result on the robust cooperative output regulation for linear uncertain multi-agent systems. *IEEE Transactions on Automatic Control*, *58*(5), 1275–1279.

Su, Y., & Huang, J. (2012). Cooperative output regulation of linear multi-agent systems. *IEEE Transactions on Automatic Control*, *57*(4), 1062–1066.

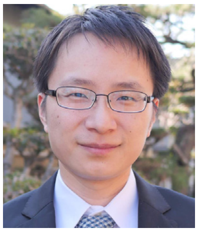
Su, Y., & Huang, J. (2013). Cooperative global output regulation of heterogeneous second-order nonlinear uncertain multi-agent systems. *Automatica*, *49*, 3345–3350.

Wang, X., Hong, Y., Huang, J., & Jiang, Z. P. (2010). A distributed control approach to a robust output regulation problem for multi-agent linear systems. *IEEE Transactions on Automatic Control*, *55*(12), 2891–2895.

Yan, Y., & Huang, J. (2017). Cooperative robust output regulation problem for discrete-time linear time-delay multi-agent systems via the distributed internal model. In *Proc. 56th IEEE conf. decision and control* (pp. 4680–4685).



Satoshi Kawamura received the B.Eng. and M.Eng. degrees in Electrical and Information Engineering from Osaka City University, Osaka, Japan, in 2017 and 2019 respectively. He is currently a member of AI Technologies R&D Department at Oki Electric Industry Co., Ltd. Mr. Kawamura's research interests include deep learning for computer vision.



Kai Cai received the B.Eng. degree in Electrical Engineering from Zhejiang University, Hangzhou, China, in 2006; the M.A.Sc. degree in Electrical and Computer Engineering from the University of Toronto, Toronto, ON, Canada, in 2008; and the Ph.D. degree in Systems Science from Tokyo Institute of Technology, Tokyo, Japan, in 2011. Since 2014 he has been an Associate Professor at Osaka City University. Dr. Cai's research interests include discrete-event systems and networked multi-agent systems. He is the co-author (with W.M. Wonham) of Supervisory Control of Discrete-Event Systems (Springer 2019) and Supervisor Localization (Springer 2016).

Masako Kishida received the B.S. and M.S.E. degrees in Aerospace Engineering and M.S. degree in Applied and Interdisciplinary Mathematics, all from University of Michigan, Ann Arbor, in 2004, 2006, and 2006, respectively and the Ph.D. degree in Mechanical Engineering from University of Illinois at Urbana-Champaign, in 2010. From 2010 to 2016, she held positions in the U.S.A., Japan, New Zealand and Germany. Since 2016, she has been an Associate Professor at National Institute of Informatics, Tokyo Japan. Her research interests include networked control systems, optimizations and uncertainty analysis. Dr. Kishida was a recipient of the Young Scientists' Prize for the Commendation of Science and Technology by MEXT, Japan in 2020.